**Medjil V1.0**

**Calibration of EDM**

**Instrument and Baseline**

**Manual**

**LANDGATE**

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The Medjil portal has been develop by Department of Land Information of Western Australia (LANDGATE) with collaboration with other jurisdictions for:

* Calibration of a Baseline against a standardised instrument
* Calibration of EDM Instruments (EDMI) against standard baseline

Regular calibration of Baselines and Instruments are required under various State and Territory regulations to ensure the distances measured are legally traceable back to the national and international standard (under *NMI Act*).

The calibration of Baseline determines new/refined distances between baseline pillars (the standard). The calibration of an EDMI provides instrument specific distance correction to be applied to measurements taken by this instrument.

The Medjil portal produces full reports for both Baseline and EDMI calibrations. Refer to specific guides for calibration procedure instructions.

This document provides reference information such as mathematical equations, procedures, algorithms, and computations adopted by the portal. The procedures and equations are based on the following references:

* “Instructions on the Verification of Electronic Distance Meters according to section 10, Weights and Measures (National Standards) Act, 1960” written by Dr. J. M. Rüeger, School of Surveying, University of New South Wales. P.O. Box 1, Kensington, N.S.W. 2033
* “Instructions on the measurements of subsidiary standards of length in the form of EDM calibration baselines using distance meters as prescribed by the National Standards Commission: written by Dr. J.M.Rüeger, School of Surveying, University of New South Wales. P.O. Box 1, Kensington, N.S.W. 2033
* “Electronic Distance Measurements” J.M.Rüeger
* Draft Verifying Authorities Handbook 2001
* IAG resolutions at it’s XXIIth General Assembly in Birmingham, 1999.
* Baseline review J.M.Rüeger 2021
* ISO ….

The Draft Verifying Authorities (VA) Handbook 2001 is intended for use by verifying authorities which are appointed under the provisions of Regulation 73 of the National Measurement Regulations 1999 in accordance with the National Measurement Act 1960 to verify reference standards of measurements under the provisions of Regulation 13, 30 and 31. The determination of the a priori standard deviations and the analysis of uncertainties of EDM measurements are based on the general guidelines within this handbook.

The minimum standards for the uncertainty of an EDM instrument calibration are described in terms of Recommendation No,8 of the Working Party of the National Standards Commission on the calibration of EDM Equipment of 1 February, 1983. All uncertainties are specified at the 95% confidence level. Users can adopt or change default uncertainties and associated traceability statement.

# Baseline Calibration

* 1. Inter-pillar distances survey
  2. Pillar offsets survey
  3. Pillar heights survey

# EDMI Calibration

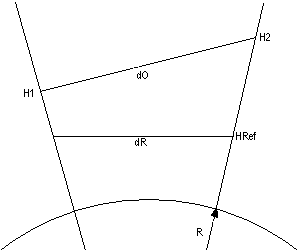
* 1. Inter-pillar distances survey

# 2. Geometric Correction

All certified distances on a calibrated baseline are on the same horizontal plane at a specified reference height and on the same vertical plane running through the first and last pillars.

The geometric corrections contain 4 components.

1. Horizontal offset correction: Correcting the baseline distances to the vertical plane running through the first and last stations.
2. Slope Correction
3. Correction to the specified reference height of the baseline.
4. Chord-to-Arc correction. This correction can be ignored. For a baseline whose length is 10 km the correction is 0.001 metres.



(2.1)

|  |  |  |
| --- | --- | --- |
| Where | dR =  dO =  ΔH =  HREF =  H1 =  H2 =  HEDM =  HREF =  O1 =  O2 =  R = | Certified horizontal distance between two pillars reduced to a reference elevation HREF  Mean observed distance between EDM instrument and reflector as corrected for atmospheric effects  H1 – H2  Reference Elevation  Elevation of EDM instrument pillar + HEDM  Elevation of reflector pillar + HREF  Height of EDM instrument above pillar  Height of reflector above pillar  Offset at pillar 1 from the straight line between the first and last pillars.  Offset at pillar 2 from the straight line between the first and last pillars.  Mean radius of curvature of the earth at the baseline location. |

To convert a certified horizontal distance between two pillars to a slope distance between the tops of these pillars the following equations can be used

(2.2)

|  |  |  |
| --- | --- | --- |
| Where | dR =  dX =  ΔH =  HREF =  H1 =  H2 =  O1 =  O2 =  R = | Certified horizontal distance between two pillars reduced to a reference elevation HREF  Slope distance between the tops of two pillars  H1 – H2  Reference Elevation  Elevation of pillar 1  Elevation of pillar 2  Offset at pillar 1 from the straight line between the first and last pillars.  Offset at pillar 2 from the straight line between the first and last pillars.  Mean radius of curvature of the earth at the baseline location. |

The geometric correction equations 2.1 and 2.2 were derived from The Australian Geodetic Datum Technical Manual, National Mapping Council of Australia, Special Publication 10 and are suitable for baselines with steep gradients.

Equation 2.2 is used in Baseline Version 5.3 to compare the observed observations with the baseline slope distances (dX). If the observation is more than a specified tolerance value from the baseline slope distance a warning message is shown. This function prevents the entry of gross errors and transcription mistakes. The observations are the measured slope distances from the EDM instrument to the reflector and may not have been corrected for atmospheric effects. Therefore the tolerance check should only be used to check for gross errors and transcription mistakes and not as an accurate comparison between the baseline slope distance and the actual observations. The default tolerance is 0.3 metres but the user can increase this value up to 5 metres.

When a new baseline is created the user is required to select a spheroid. The default spheroid is GRS80, which had a semi-major axis of 6378160. The baseline software uses the semi-major axis of the selected spheroid as the value for R in equations 2.1 and 2.2. This value is adequate for baselines less than 15 km in length anywhere in Australia. The error caused by the approximation of R in equations 2.1 and 2.2 is less than 0.001 metres over a length of 15 km. For a more accurate determination of R use the following equations derived from “The Australian Geodetic Datum Technical Manual, National Mapping Council of Australia, Special Publication 10 (Chapter 3.2)”

(2.3)

(2.4)

(2.5)

(2.6)

|  |  |  |
| --- | --- | --- |
| Where | p =  v =  a =  f =  Q = | Radius of curvature of the spheroid in meridian  Radius of curvature of the spheroid in prime vertical  Semi-major axis of the spheroid  Flattening of the spheroid  Latitude of the baseline location |

The following algorithms (2.7 and 2.8) have traditionally been used to reduce very long lines to the reference elevation. However these algorithms are not suitable for baselines with large height differences and horizontal offsets between the pillars. These algorithms were used in the superseded versions of the Baseline software. (Prior to Version 5.3)

 (2.7)

 (2.8)

|  |  |  |
| --- | --- | --- |
| Where | DH =  ER =  D =  ΔH =  H1 =  H2 =  HEDM =  HREF =  R =  DSKEW=  O1=  O2= | Horizontal distance reduced to a reference elevation ER  Reference elevation  Mean slope distance between EDM instrument and reflector, corrected for horizontal offsets to the straight line between the first and last pillars  (H2+HEDM) – (H1+HREF)  Elevation of EDM instrument pillar  Elevation of reflector pillar  Height of EDM instrument above pillar  Height of reflector above pillar  Mean radius of curvature of the earth at the baseline location.  Observed Distance not corrected for the horizontal offsets at the pillars.  Horizontal offset at pillar 1 to the straight line between the first and last pillars.  Horizontal offset at pillar 2 to the straight line between the first and last pillars. |

There is no difference in the geometric correction as calculated by both algorithms (equation 2.1 and 2.7) provided that the slope of the baseline is less than 15 degrees.

The following table contains the errors in the geometric reduction if equation 2.7 is used when the slope of the baseline exceeds 15 degrees.

|  |  |
| --- | --- |
| Slope  (degrees) | Error in the geometric correction over a horizontal baseline distance of 1000 metres (Baseline Version 5.1) |
| 15 | < 0.001 metres |
| 20 | 0.009 metres |
| 25 | 0.050 metres |
| 30 | 0.213 metres |

# 3. Atmospheric correction

## 3.1 First velocity correction (K) for EDM instruments.

(3.1)

|  |  |  |
| --- | --- | --- |
| Where | K =  P =  t =  d =  e =  C =  nREF =  D =  nG= | First velocity correction in metres  Pressure in millibars/hectapascals  Dry temperature in degrees Celsius  Distance in metres  Partial water pressure in millibars/hectapascals (equation 3.9)  (NREF – 1) 106  Reference refractive index (equation 3.2)  (nG – 1) 106 (273.15 / 1013.25)  Group refractive index of atmosphere for standard conditions (equation 3.4) |

## 3.2 Reference Refractive Index (nREF)

The reference refractive index is instrument specific.

(3.2)

|  |  |  |
| --- | --- | --- |
| Where | CO =  MOD =  fMOD =  U = | Velocity of light in a vacuum = 299 792 458 m/s  Constant modulation wavelength of the fine measurements for which the instrument is designed.  Constant modulation frequency of the fine measurements  The exact half of MOD, called the unit length of the instrument. |

The velocity of light in a vacuum is fixed at 299792458 m/s by the 1983 definition of the SI metre.

The unit length of the instrument (U) and the constant modulation frequency of the fine measurements (fMOD) are instrument specific and are generally provided by the manufacturer.

Appendix E contains a list of U and fMOD for several electro-optical EDM instruments. Currently 10 metres is the most common unit length. The reference refractive index nREF of an instrument is fixed by the manufacturer by adopting a suitable unit length and by adjusting the main oscillator to such a modulation frequency so that nREF corresponds more or less to an average refractive index encountered under field conditions.

## 3.3 Group refractive Index of light in the atmosphere.

In electro-optical EDM, the refractive index is dependent on the wavelength of the visible or infrared radiation. Different frequencies have the same propagation velocity in a vacuum, but not in air because of the interference occurrence between the different frequencies. The signal resulting from the sum of all frequencies will have the so-called group velocity, which is always smaller than the phase velocities of its individual frequencies.

The International Association of Geodesy (IAG) resolved in 1999 at its XXIIth General Assembly in Birmingham that the refractive index in electronic distance measurements (nL) can be calculated using the computer procedure published by Ciddor & Hill in Applied Optics(1999, Vol.38, N0.9, 1663-1667) and Ciddor in Applied Optics (1996, Vol.35, No.9, 1566-1573).

The following closed formulae can be adopted for the computation of the group refractive index in air for electronic distance measurements to within 1 ppm with visible and near infrared waves in the atmosphere:

(3.3)

|  |  |  |
| --- | --- | --- |
| Where | NL =  nL =  NG =  t=  p =  e = | Group refractivity of visible and near infrared waves in ambient moist air. Valid for atmospheric conditions described by t, p and e.  Corresponding group refractive index.  Group refractivity index for standard conditions (visible light in dry air at 0 oC, 1013.25 hPa, partial water vapour pressure = 0 and the air contains 0.0375 % CO2). See Equation 3.4.  Dry bulb temperature of air (oC).  Atmospheric pressure in hectapascals  Partial water vapour pressure (hPa) |

(3.4)

These closed formulae (equations 3.3 and 3.4) deviate less than 0.25 ppm from the accurate formulae between -30ºC and +45ºC, at 1000 hPa pressure, 100% relative humidity (without condensation) and for wavelengths between 650 nm and 850 nm. The 1 ppm stated before makes some allowance for anomalous refractivity and the uncertainty in the determination of the atmospheric parameters.

The above equations (3.3 and 3.4) were recommended by the International Association of Geodesy (IAG) at its XXIIth General Assembly in Birmingham, 1999 and have been used in the software **”Baseline Version 5”**.

For high precision EDM at all visible and infrared wavelengths, you could also consider using the following formulae (Equations 3.5 to 3.8 - Owens 1967).

(3.5)

(3.6)

(3.7)

(3.8)

|  |  |
| --- | --- |
| Where P =  PS =  PW =  PS =  T =  t = | total atmospheric pressure (hPa)  P – PW (hPa)  partial water pressure (hPa)  partial pressure of dry air containing 0.03% CO2 in standard air (hPa)  absolute temperature (K) = 273.15+t  temperature (ºC) |

These formulae are valid from the farthermost infrared down to the ultraviolet (from 1960 nm to 230 nm) These equations provide an accuracy of two parts in 108.

These equations (3.5 to 3.8) were used in previous versions of the software **“Baseline”**. However these equations were not mentioned by the International Association of Geodesy (IAG) at its XXIIth General Assembly in Birmingham, 1999 and have therefore not been used in the latest version of the software **”Baseline Version 5”**.

## 3.4 Partial water vapour pressure

The partial water vapour pressure can either be derived from dry bulb and wet bulb readings of an aspiration psychrometer or can be derived from known values of relative humidity. Relative humidity can also be measured with humidity sensors or hygrometers or may be available from daily weather information. The accuracy of Hygrometers is about 3%, which is not always accurate enough for EDM purposes. Hence only psychrometers or Humidity sensors should be used.

### 3.4.1 Psychrometers with Mercury-in-Glass Thermometers

A psychrometer comprises a dry and wet thermometer and a small spring driven fan. Any fully ventilated psychrometer is acceptable for calibration. The user should arrange to have the thermometers calibrated at a NATA accredited laboratory. The covering for the wet element is replaced if necessary and the reassembled instrument is compared with a reference psychrometer under one set of conditions to test for any significant departure of the psychrometer readings from the accepted values. If the results of the psychrometer intercomparison are satisfactory, the report states this fact. Psychrometers with mercury-in glass thermometers used for the calibration of EDM instruments should be accurate to 0.2oC

### 3.4.2 Electronic Psychrometers

Electronic psychrometers contain electronic thermometers and usually incorporate a microprocessor so that the relative humidity can be calculated from the dry and wet element temperatures and displayed. In order to calibrate these instruments, it must be possible to calibrate the thermometer elements, while they are still attached to the electronic readout. Also, it must be possible to read both the dry and wet element temperatures.

If the relative humidity is displayed, it is usually possible to provide a table of corrections to be applied to the displayed relative humidity readings.

An electronic psychrometer is calibrated in a similar way toa Mercury-in Glass psychrometers, involving a full calibration in which the thermometers are checked and the psychrometer correction is calculated. The thermometer elements should then be checked yearly and a full calibration repeated if the thermometer corrections drift. Electronic psychrometers used for the calibration of EDM instruments should be accurate to 0.2oC

### 3.4.3 Humidity Sensors

The accuracy of humidity sensors is typically specified as **2%** relative humidity(R.H.) between 0 and 80% R.H. and **3%** between 80 and 100% R.H. The temperature sensitivity of the humidity reading is typically **0.05% R.H./oC**. Annual calibration is necessary to maintain these accuracies.

### 3.4.4 Computation of partial water vapor pressure from psychrometer readings

 (3.11)

|  |  |  |
| --- | --- | --- |
| Where | e =  E’W =  p=  t =  t’ = | Partial water vapour pressure in hectapascals  Saturation water vapour pressure for temperature t (wet bulb temperature) in hectapascals over water.  Atmospheric pressure in hectapascals  Dry temperature in degrees Celsius.  Wet temperature in degrees Celsius. |

This formula is accurate to 1% and valid for t > 0oC and t’ > 0oC. In the case of a frozen wet bulb wick, the following equation must be used.

 (3.12)

Where E’ICE is the saturation water vapour pressure over ice at the temperature t’ (wet bulb temperature) in hPa.

 (3.13)

 (3.14)

The maximum errors of equation 3.13 occur at -20oC and +50oC and amounts to 0.20%. The maximum errors of equation 3.14 do not exceed 0.02% at -50oC and 0oC.

### 3.4.5 Computation for partial vapour pressure from relative humidity

(3.15)

|  |  |  |
| --- | --- | --- |
| Where | e =  E =  h= | Partial water vapour pressure in hectapascals  Saturation water vapour pressure (hPa) at the dry bulb temperature (equation 3.16).  Relative humidity in percent. |

(3.16)

|  |  |  |
| --- | --- | --- |
| Where | p =  t = | Atmospheric pressure in hectapascals  Dry temperature in degrees Celsius |

# 4. Estimating the uncertainties

The following uncertainties affect the least squares estimated EDM instrument corrections when calibrating any instrument against a standard baseline.

* Uncertainty of the horizontal pillar intervals of a standard baseline.
* Uncertainty of the horizontal offsets at the baseline pillars.
* Uncertainty of the height differences between the pillars at the baseline.
* Uncertainty due to the meteorological observations (temperature, pressure and humidity).
* Uncertainty due to centring the EDM instrument and the reflector.
* Uncertainty of the EDM distance readings for each measured interval between the baseline pillars.

The following uncertainties effect the least squares estimated horizontal distances between the pillars of a baseline when calibrating this baseline against a prescribed EDM instrument.

* Uncertainty of the prescribed EDM instrument.
* Uncertainty of the horizontal offsets at the baseline pillars.
* Uncertainty of the height differences between the pillars at the baseline.
* Uncertainty due to the meteorological observations (temperature, pressure and humidity).
* Uncertainty due to centring the EDM instrument and the reflector.
* Uncertainty of the EDM distance readings for each measured interval between the baseline pillars.

## 4.1 Uncertainties of the horizontal pillar intervals of standard baselines in Western Australia

**Table 4.1** refers to the certificate of Verification (Regulation 13) of the Weights and Measures (National Standards) regulation pertaining to the standardisation of the monumented EDM baseline in Kalgoorlie, for the verification date 11th September, 2001.

|  |  |  |  |
| --- | --- | --- | --- |
| Interval  (metres) | | Horizontal  Distance  (metres) | Standard  Deviation  (mm) |
| From | To |
| 1 | 2 | 4.9912 | 0.45 |
| 1 | 3 | 9.9951 | 0.43 |
| 1 | 4 | 71.9865 | 0.43 |
| 1 | 5 | 133.8974 | 0.48 |
| 1 | 6 | 235.8418 | 0.52 |
| 1 | 7 | 377.8853 | 0.58 |
| 1 | 8 | 599.8503 | 0.64 |
| Table 4.1 | | | |

The above table only provides standard deviations of the baseline intervals from pillar 1. However during a calibration of an EDM instrument a baseline interval between any two pillars may be used. An a priori standard deviation must computed for each baseline interval used in a calibration of an EDM instrument. These standard deviations can be obtained from the Variance-Covariance matrix of the adjusted baseline intervals from the most recent baseline calibration. However this Variance-Covariance matrix may no longer be available and an approximate solution must then be used:

Assuming a linear relationship of the standard deviation with respect to the baseline interval distance the standard deviation (sB)of any Kalgoorlie baseline interval can be approximated using a linear regression solution and the values in table 1:

 (4.1)

Where sB is the standard deviation of a baseline interval

Using the values in table 1 or 2 a linear regression solution is used to solve equation 4.1

 (4.2)

 (4.3)

Let X = certified distance of each baseline interval in metres

Y = standard deviation of each baseline interval in metres

n = number of baseline intervals

**Example**: calculate (sB) of the Kalgoorlie baseline using table 1 and equation 4.1 to 4.3

|  |  |  |  |
| --- | --- | --- | --- |
| **Baseline Interval**  **X** | **Std Dev**  **Y** | **XY** | **X2** |
| 4.9912  9.9951  71.9865  133.8974  235.8418  377.8853  599.8503 | 0.00045  0.00043  0.00043  0.00047  0.00052  0.00058  0.00064 | 0.002246  0.004298  0.030954  0.062932  0.122638  0.219173  0.383904 | 24.9121  99.9020  5182.0562  17928.5137  55621.3546  142797.3000  359820.3824 |
|  X= 1434.4476 |  Y= 0.00352 |  XY= 0.826145 |  X2= 581474.4210 |

Number of observations (n) = 7

Using equation 4.2

****

Using equation 4.3



The effect of the uncertainty of the Kalgoorlie baseline intervals on the calibration of an EDM instrument is:

**Standard deviation of a Kalgoorlie baseline interval (sB)**  **= 0.43 mm + 0.365 ppm**

Using a coverage factor of 2 the uncertainty at the 95% confidence level of the Kalgoorlie baseline interval (sB) can be calculated.

**Uncertainty of a Kalgoorlie baseline interval (95%) = 0.86 mm + 0.73 ppm**

**Table 4.2** refers to the certificate of Verification (Regulation 13) of the Weights and Measures (National Standards) regulation pertaining to the standardisation of the monumented EDM baseline at Curtin University, Perth, for the verification date 11th September, 2001.

|  |  |  |  |
| --- | --- | --- | --- |
| Interval  (metres) | | Horizontal  Distance  (metres) | Standard  Deviation  (mm) |
| From | To |
| 1 | 2 | 42.4969 | 0.33 |
| 1 | 3 | 44.9953 | 0.33 |
| 1 | 4 | 52.5960 | 0.33 |
| 1 | 5 | 187.5438 | 0.33 |
| 1 | 6 | 321.8094 | 0.35 |
| 1 | 7 | 472.4830 | 0.36 |
| 1 | 8 | 493.5312 | 0.37 |
| 1 | 9 | 514.5397 | 0.39 |
| 1 | 10 | 535.5319 | 0.42 |
| 1 | 11 | 559.5201 | 0.44 |
| 1 | 12 | 582.4873 | 0.46 |
| Table 4.2 | | | |

The effect of the uncertainty of the Curtin baseline intervals on the calibration of an EDM instrument is:

**Standard deviation (sB) = 0.30 mm + 0.19 ppm**

Using a coverage factor of 2 the uncertainty at the 95% confidence level of the Curtin baseline interval (sB) can be calculated.

**Uncertainty of a Curtin baseline interval (95%) = 0.61 mm + 0.38 ppm**

## Uncertainty of the horizontal offsets at the baseline pillars.

The effect of the offset errors on the distance can be analysed by differentiating equation 2.1.

 (4.4)

Where D = horizontal distance.

Oi = Horizontal offset at pillar i from the straight line between the first and last pillars.

Oj = Horizontal offset at pillar j from the straight line between the first and last pillars.

dD = Differential of distance

dOi = Differential of the offset at pillar i

dOj = Differential of the offset at pillar j

Let sOffset = standard deviation of the offset measurements at both pillar i and j

and sO = The effect of sOffset on the standard deviation of the distance between pillar i and j.

 (4.5)

The offset distances of the pillars at the Curtin and Kalgoorlie baselines are very small

|  |  |
| --- | --- |
| Pillar | Offset  (metres) |
| 1  2  3  4  5  6  7  8  9  10  11  12 | 0.0000  -0.0020  0.0000  -0.0080  -0.0185  -0.0075  0.0055  -0.0030  0.0060  -0.0120  -0.0060  0.0000 |

**Table 4.3:** Pillar offsets at the Curtin baseline (2001 Calibration).

**Example:**

Using the table 4.3 calculate the effect of the offset uncertainty on the distance between pillar 7 and 8.

Let sOffset = 2mm =Standard deviation of the offset measurements at pillars 7 and 8.

D = 21.0482 metres = 21048.2 mm = distance between pillars 7 and 8.

O7 = +0.0055 metres = +5.5 mm

O8 = -0.0030 metres = -3 mm

Using equation 4.5

Standard deviation of the distance =

The offsets at the pillars at both the Curtin and Kalgoorlie baselines are very small. Hence the effect of the offset uncertainty on the distance is insignificant.

The following uncertainties at the 95% confidence level have been adopted for the Curtin and Kalgoorlie baselines:

|  |  |  |
| --- | --- | --- |
| Uncertainty Component | Uncertainty for  EDM Instrument Calibration | Uncertainty for  Baseline Calibration |
| Pillar Offset | 1.0 mm | 1. 0 mm |

## 4.3 Uncertainty of the height differences between the pillar at the baseline.

The effect of the height difference on the horizontal distance can be determined by differentiating equation 2.2

 (4.6)

|  |  |  |
| --- | --- | --- |
| Where | DH =  H =  Hi =  Hj =  HEDM =  HREF =  dDH =  dH = | Horizontal distance  (Hi+HEDM) – (Hj+HREF)  Elevation of EDM instrument pillar  Elevation of reflector pillar  Height of EDM instrument above pillar  Height of reflector above pillar  Differential of DH.  Differentials of H |

Let sL = standard deviation of the distance caused by the uncertainties in measuring the height difference.

 (4.7)

|  |  |  |
| --- | --- | --- |
| Where | H =  HEDM =  HREF = | Standard deviation of the height difference between the EDM and reflector pillars.  Standard deviation of height of EDM instrument above pillar  Standard deviation of height of reflector above pillar |

The height differences between the pillars at the Curtin and Kalgoorlie baselines are small.

|  |  |
| --- | --- |
| Pillar | Elevation  (metres) |
| 1  2  3  4  5  6  7  8  9  10  11  12 | 10.394010.3260  10.2290  10.2160  9.9340 10.355010.3090 10.4330 10.7280 11.1460 11.6050  11.8250 |

**Table 4.4:** Pillar elevations at the Curtin baseline (2001 Calibration)

**Example:**

Using the above table (4.4) calculate the effect of the height uncertainty on the distance between pillar 9 and 10.

Let H = 1 mm HEDM = 2mm, HREF = 2 mm

DH = 20.9922 metres = 20992.2mm and H = 0.418 metres = 418 mm

Using equation 4.7



The following uncertainties at the 95% confidence level have been adopted for the Curtin and Kalgoorlie baselines:

|  |  |  |
| --- | --- | --- |
| Uncertainty Component | Uncertainty for  EDM Instrument Calibration | Uncertainty for  Baseline Calibration |
| Height of EDM above pillar | 2.0 mm | 1.0 mm |
| Height of reflector above pillar | 2.0 mm | 1.0 mm |
| Height difference between pillars | 1.0 mm | 1.0 mm |

## Uncertainty due to meteorological observations (temperature, pressure and humidity).

The effect of errors in measurements of atmospheric pressure, temperature and partial water vapour pressure on the group refractive index (nL) can be analysed by differentiating equation 3.3.

 (4.8)

 (4.9)

 (4.10)

 (4.11)

|  |  |  |
| --- | --- | --- |
| Where | DnL =  Dt =  Dp =  De = | Differential of the group refractive index of light  Differential of the temperature t (oC)  Differential of the pressure p (hPa)  Differential of the partial water vapour pressure e (hPa) |

For example for a temperature of 15 oC, a pressure of 1007hPa, a partial water vapour pressure of 13hPa and a group refractive index nG of 1.0003045 yields

 (4.12)

The significance of errors in the meteorological observations on the EDM distance can be summarised as follows:

* An error in temperature of 1 oC affects the refractive index and distance by 1ppm
* An error in pressure of 1hPa affects the refractive index and distance by 0.3ppm
* An error in the partial water vapour pressure (e) of 1mb affects the refractive index and distance by 0.04 ppm and therefore e need not be known very accurately.

 (4.13)

|  |  |
| --- | --- |
|  | (4.14) |
|  | (4.15) |
|  | (4.16) |

Equation 4.13 can be simplified as follows:

 (4.17)

The Humidity has little effect on an EDM distance and se is frequently omitted from equation 4.13 and 4.17.

The standard deviation of the partial water vapour (se ) can be derived from equation 3.15

 (4.18)

where Humidity = standard deviation of the humidity in percentage.

Temperature and pressure are critical for the determination of the refractive index. Temperature and pressure may be measured accurately at both terminals of a line and the refractive index calculated for both terminals and the mean taken. All meteorological equipment is to be shaded by an umbrella in sunny and rainy conditions.

The following uncertainties at the 95% confidence level have been estimated for the calibration on the Curtin or Kalgoorlie baseline. A coverage factor of 2 has been used.

|  |  |  |
| --- | --- | --- |
| Meteorological Uncertainty components | Uncertainty for  EDM Instrument  Calibration | Uncertainty for  Baseline  calibration |
| Temperature | 1.5 oC | 0.8 oC |
| Barometric Pressure | 2 hPa | 1 hPa |
| Humidity (Percentage) | 6% | 3% |
| Total Atmospheric Uncertainty (using equation 4.13) | 2.1 ppm | 1.0 ppm |

## 4.5 Uncertainty of centring the EDM instrument and reflector

 (4.19)

|  |  |  |
| --- | --- | --- |
| Where | sC =  EDM =  REF = | Standard deviation caused by uncertainties in centring the EDM instrument and reflector  Standard deviation of centring the EDM instrument  Standard deviation of centring the reflector |

**Example:**

The standard deviation of centring the EDM instrument (EDM) = 0.5 mm.

The standard deviation of centring the reflector (REF) = 0.5 mm.

Hence 

The following a uncertainties at the 95% confidence level have been adopted for calibrations on the Curtin and Kalgoorlie baselines:

|  |  |  |
| --- | --- | --- |
| Uncertainty components | Uncertainty  EDM Instrument  Calibration | Uncertainty  Baseline  Calibration |
| Centring EDM instrument | 0.20 mm | 0.10 mm |
| Centring Reflector | 0.20 mm | 0.10 mm |
| Total Centring Uncertainty (using equation 4.19) | 0.28 mm | 0.14 mm |

## 4.6 Uncertainty of the EDM Distance readings.

A priori standard deviations of the EDM distance measurements (sS) can be obtained using the following methods.

* Adopting the manufacturer’s specifications. These specifications are totally unsuitable for estimating the standard deviations of distance measurements on a baseline.
* Adopting a value based on previous least square solutions on the baseline with similar instruments. The program “BASELINE” allows the user to enter an a priori estimation of the standard deviation. The least squares report will indicate whether or not a reliable estimation has been carried out. This method is not recommended and should only used when there are not enough observations to provide a reliable standard deviation for each line.
* A good estimate of the a priori standard deviations of the distance readings can be obtained from the actual measurements provided that there are enough measurements and that the correct field procedures and computation methods are applied. For each measured line the mean of all distance measurements (at least 4 measurements) is computed as well as the standard deviation (sS). The equation used in determining the standard deviation of the EDM measurements for a line is derived from the VA handbook, Section 3.5

 (4.20)

\_

Where D = Mean of all measurements for one baseline interval

D= Single measurement of the same baseline interval

n = number of measurements

A linear regression solution is used to resolve an a priori standard deviation (sx) of the measured line caused by random errors in the distance measurements of all lines used on the baseline. At this stage, errors in centring, levelling and meteorological measurements are not included. sx is expressed as a standard deviation for an additive constant and a scale correction

 (4.21)

Let X = mean distance of each measured line

Y = (sS) = standard deviation of the EDM measurements for a line. (equation 4.19)

n = number of measured lines

 (4.22)

 (4.23)

## 4.7 Uncertainty of a prescribed EDM instrument used for calibrating a standard baseline

The EDM instrument “Kern Mekometer ME 5000 Serial Number 3507090” and reflector “Kern ME 500 Serial Number 375629” were used in the calibration of the Curtin and Kalgoorlie baselines in September 2003.

The frequency counter of the Mekometer ME 5000 was calibrated by the CSIRO on the 15th October 2002 against the National Frequency Standard and found to be:

9 999 998.60 Hz with an uncertainty of 0.7Hz at the 95% confidence level

The frequency counter should have an accuracy specification of 1 part in 108 (1Hz) between 00 and 500C. (Refer to Electronic Distance Measurements by J.M.Rueger)

According to the manufacturer the standard deviation (si) of the Kern Mekometer ME 5000 is:

**si = X(mm) + Y(ppm) = 0.2 mm + 0.2 ppm** (4.24)

However the manufacturer’s specifications are generally unsuitable for the estimation of the precision of distance measurements on baselines. Therefore caution should be used if you want to include these specifications in the computation of the uncertainties.

## Combining all uncertainties

The a priori standard deviation of each measured line

 (4.25)

For the calibration of an EDM instrument the following equation is used to compute C in equation 4.25:

C2  = A2 + P2 +s2O + s2L+ s2C  (4.26)

For the calibration of a baseline against a prescribed EDM instrument the following equation is used to compute C in equation 4.25

:

C2  = A2 + X2 + s2O + s2L+ s2C (4.27)

|  |  |  |
| --- | --- | --- |
| Where | A =  P =  X =  SO =  sC =  sL = | Constant component of the standard deviation of the EDM distance readings. Obtained by linear regression using equation 4.22  Constant component of the standard deviation of the horizontal pillar intervals. Obtained by linear regression using equation 4.2  Constant component of the standard deviation of the prescribed instrument used in the calibration of the baseline. (equation 4.24). In this case the manufacturer’s specifications are used.  Constant component of the standard deviation of the horizontal pillar interval. Obtained by linear regression using equation 4.2  Standard deviation of the pillar offset (equation 4.5)  Standard deviation of centring EDM equipment or reflector on pillar. (equation 4.19)  Standard deviation of the height difference (equation 4.7) |

For the calibration of an EDM instrument the following equation is used to compute D in equation 4.25

D2  = B2 + Q2 + s2A (4.28)

For the calibration of a baseline against a prescribed EDM instrument the following equation is used to compute D in equation 4.25

D2  = B2 + Y2 + s2A (4.29)

|  |  |  |
| --- | --- | --- |
| Where | B =  Q =  Y =  sA = | Scale component of the standard deviation of the EDM distance readings. Obtained by linear regression using equation 4.23  Scale component of the standard deviation of the horizontal pillar interval. Obtained by linear regression using equation 4.3  Scale component of the standard deviation of the prescribed instrument used in the calibration of the baseline. (equation 4.24). In this case the manufacturer’s specifications are used.  Standard deviation of the meteorological observations. Obtained from equation 4.13 |

## 4.9 Summary of Uncertainties used at the Curtin and Kalgoorlie baseline

The following uncertainties at the 95% confidence level have been adopted for calibrations on the Curtin and Kalgoorlie baselines.

|  |  |  |
| --- | --- | --- |
| Uncertainty Component | Uncertainties for  EDM Instrument Calibration | Uncertainties for  Baseline Calibration |
| EDM distance readings | Based on actual observations | Based on actual observations |
| Prescribed EDM instrument (Mekometer ME5000) | N/A | 0.4 mm+ 0.4 ppm |
| Baseline Interval (Kalgoorlie 2001) | 0.86 mm + 0.73 ppm | N/A |
| Baseline Interval (Curtin 2001) | 0.61mm + 0.38 ppm | N/A |
| Height of EDM above pillar | 2.0 mm | 1.0 mm |
| Height of reflector above pillar | 2.0 mm | 1.0 mm |
| Height difference between pillars | 1.0 mm | 1.0 mm |
| Temperature | 1.5 oC | 0.8 oC |
| Barometric Pressure | 2 hPa | 1 hPa |
| Humidity (Percentage) | 6% | 3% |
| Centring EDM instrument | 0.2 mm | 0.1 mm |
| Centring Reflector | 0.2 mm | 0.1 mm |
| Pillar Offset | 1.0 mm | 1.0 mm |
| **Table 4.5** | | |

The uncertainties of the Pillar offsets and height differences have an insignificant impact on the uncertainties of the observed distances on the Curtin and Kalgoorlie baselines.

The uncertainties in table 4.5 have been derived as follows:

* The estimated uncertainties of the Temperature, Barometric Pressure and Centring have been derived from an analysis of the calibration results from several previous Baseline and EDM instrument calibrations on the Curtin and Kalgoorlie baselines.
* Based on the accuracy specifications of the equipment.
* Based on the accuracy statements in “Electronics Distance Measurement” by J.M,Rueger.

A coverage factor of 2 has been used for converting the uncertainties into a priori standard deviations for the least squares computations

## 4.10 Example of calculating the a priori standard deviation of a measured line.

Calculate the a priori standard deviation of the distance between pillars 3 and 5 when calibrating an EDM instrument on Curtin Baseline.

|  |  |  |  |
| --- | --- | --- | --- |
| A Priori Standard deviation Component | Symbol | A priori  Std Dev | Comments |
| Mean temperature  Mean pressure  Partial water vapour pressure  Centring EDM instrument  Centring Reflector  Pillar offset  Height Difference between pillars  Height of EDM instrument above pillar  Height of reflector above pillar  EDM distance readings  Certified baseline interval | sT  sP  se  CEDM CREF  sOffset  H  HEDM  HEDM  sS  sB | 1.0oC  1 hPa  1 hPa  0.5 mm  0.5 mm  2 mm  0.5 mm  0.5 mm  0.5 mm  Table 6  Section 4.1 | At both the instrument and reflector pillars.  Y column in table 6 (derived from equation 4.21)  Pmm + Qppm = 0.30 mm + 0.19 ppm |

The a priori standard deviations in the above table are derived from the uncertainties of the Curtin Baseline in table 4.5. A coverage factor of 2 has been used for converting these uncertainties to standard deviations.

|  |  |  |
| --- | --- | --- |
| Auxiliary Observations | Symbol | Value |
| Mean temperature  Mean pressure  Partial water vapour pressure Group refractive index  Horizontal distance between pillar 3 and 5  Offset at pillar 3  Offset at pillar 5  Height difference between pillars 3 and 5 | T  p  e  nG  DH  O3  O5  H | 20oC  1010 hPa  13 hPa  1.0003045  142.5485 metres  0 mm  -18.5 mm  0.2950 metres |

In this example the standard deviation of the single distance observation (Y) for each measured line has already been calculated as per section 4.6 and equation 4.20.

A linear regression solution (equations 4.22 and 4.23) is used to resolve the a priori standard deviation of the measured line caused by random errors in the distance measurements (Section 4.6)

|  |  |  |  |
| --- | --- | --- | --- |
| **Mean Distance**  **X** | **Std Dev**  **Y** | **XY** | **X2** |
| 145.0476  279.3125  429.9893  451.0345  472.0435  493.0300  517.0280  514.5235  490.5365  469.5448  448.5380  427.4890  276.8140  142.5493 | 0.0019  0.0013  0.0017  0.0013  0.0024  0.0022  0.0016  0.0025  0.0017  0.0021  0.0034  0.0034  0.0008  0.0005 | 0.2756  0.3631  0.7310  0.5863  1.1329  1.0847  0.8272  1.2863  0.8339  0.9860  1.5250  1.4535  0.2215  0.0713 | 21038.8063  78015.4727  184890.7981  203432.1202  222825.0659  243078.5809  267317.9528  264734.4321  240626.0578  220472.3192  201186.3374  182746.8451  76625.9906  20320.3029 |
|  X= 5557.4805 |  Y= 0.0268 |  XY= 11.3783 |  X2= 2427311.0820 |

Table 4.6

Number of observations (n) = 14

Using equation 4.22



Using equation 4.23



Calculate standard deviation in ppm caused by the meteorological uncertainties.

Using equation 4.13





Derive the standard deviation of the certified distance between pillar 3 and 5

sB = 0.33 mm (Derived from table 2)

Calculate standard deviation caused by the offsets at pillars 3 and 5

Using equation 4.5



Calculate standard deviation caused by height difference between pillars 3 and 5

Using equation 4.7



Calculate standard deviation caused by centring the EDM instrument and reflector

Use equation 4.19



Using equation 4.26

C2  = A2 + P2 + s2O + s2L+ s2C



Using equation 4.28



Hence the a priori standard deviation of the measured line is:

**(sD) = (Cmm + Dppm) = 0.97 mm + 3.45 ppm**

# 5. Least squares solutions of EDM corrections

## 5.1 Systematic errors in EDM instruments

There are three distinct systematic errors, which may occur in EDM instruments.

* Zero constant or index error
* Scale error
* Cyclic or short periodic error

**Zero constant or index error (a0)**

All distances measured by a particular EDM instrument and reflector combination are subject to a constant error caused by three factors:

* Electrical delays, geometrical detours and eccentricities in the instruments.
* Differences between the electronic centre and the mechanical centre of the instrument.
* Differences between the optical and mechanical centres of the reflector.
* This error may vary with changes of reflector, or after jolts, or with different instrument mountings. It is an algebraic constant to be applied directly to every measurement.

**Scale Error (a1)**

Scale errors are proportional to the length of the line measured and is caused by:

* Internal frequency errors, including those caused by external temperature and instrument “warm up” effects.
* Errors of measured temperature, pressure and humidity affect the velocity of the light.
* Non-homogeneous emissions/reception patterns from the emitting and receiving diodes (phase inhomogeneities).

**Cyclic Errors (b11, b12 b21, b22)**

The precision of an EDM instrument is dependent on the precision of the internal phase measurements. Unwanted interference through electronic/optical cross talk or multi-path effects of the transmitted signal on to the received signal causes cyclic errors. The major form of the cyclic error is sinusoidal with a wavelength equal to the unit length of the instrument. The unit length is the scale on which the EDM instrument measures the distance and is derived from the fine measuring frequency. Unit length is equal to one half of the modulation wavelength.

## 5.2 Mathematical model for the calibration of EDM instruments on a baseline

IC = a0 + a1 D + sin (2D/U) b11 + cos(2D/U) b12 + sin(4D/U) b21+ cos(4D/U) b22 (5.1)

|  |  |  |
| --- | --- | --- |
| Where | IC =  a0 =  a1 =  D =  U = | Instrument additive constant (also referred to as zero error or index correction  Zero error  Scale error  Measured distance between two pillars.  Unit length of an EDM instrument |
| b11, b12, b21 and b22 are the cyclic error parameters | |

The angular values (sin and cos values) are expressed in radians, to convert the angular values to decimal degrees multiply by 180/. The mathematical model for the EDM instrument correction contains a cyclic error component, which is based on two different wavelength cycle errors, Their wavelengths being U and U/2.

The amplitude and phase of each wavelength cyclic error can be derived from equations 5.2 and 5.3.

 (5.2)

 (5.3)

The most common unit length is 10 metres. Equation 5.1 can be simplified for EDM instruments having a unit length of 10 metres

IC = a0 + a1 D + sin (36D) b11 + cos(36D) b12 + sin(72D) b21+ cos(72D) b22 (5.4)

Equation 5.4 can only be applied for EDM instruments containing a 10 metre unit length. The angular values (sin and cos values) in this equation are expressed in decimal degrees.

## 5.3 Observation equations

Assume that n distances are measured with an EDM instrument on a baseline. An observation equation of the following form for each measured distance can be derived. The angular values (sin and cos values) are expressed in radians.

vj = a0 + a1 D’j + sin (2 Dj /U) b11 + cos(2 Dj /U) b12 + sin(4 Dj /U) b21+ cos(4 Dj /U) b22 + (D’j - Dj))

(5.5)

Certified

|  |  |  |
| --- | --- | --- |
| Where | Dj (j = 1,2,……..,n) is the jth observed distance  D’j is the true value of Dj (Fixed distance between pillars of baseline) | |
|  | vj = | Residual is the difference between fixed pillar interval and the adjusted observed distance. The observed distance has been corrected by the least squares estimated instrument correction |

The observation equations for all distances measured on a baseline can be further expressed in a matrix form as

**V = AX + W** (5.6)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Where A = | 1  1  …  1 | D1  D2  …  Dn | Sin (2 D1 /U)  Sin (2 D2 /U)  …  sin (2 Dn /U) | cos (2 D1 /U)  cos (2 D2 /U)  …  cos (2 Dn /U) | sin (4 D1 /U)  sin (4 D2 /U)  …  sin (4 Dn /U) | cos (4 D1 /U)  cos (4 D2 /U)  …  cos(4 Dn /U) |

(5.7)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| V = | v1  v2  …  vn | X= | a0  a1  b11  b12  b21  b22 | W = | D’1- D1  D’2- D2  …  D’n- Dn |

(5.8)

A weight matrix can be determined for the observed distances according to their accuracy

 (5.9)

Where 20 is the a priori variance factor and CL is the diagonal matrix containing the a priori variances of the measured lines. 20 is used for scaling the input variances of the observations prior to the least squares solution. Usually 20 = 1 is chosen, which means that the input variances are not scaled.

if 20 = 1 is chosen, the weight of the jth distance is

pj = 1 /SD2 (5.10)

Where sD = a priori standard deviation of the measured jth distance

Refer to equation 4.20 (section 4) for the derivation of the standard deviation of a measured line.

## 5.4 Least squares solution of the parameters

It is well known that the least squares solution of the observation equations is

**X = ( ATPA)-1 ATPW**  (5.11)

Where X = least squares estimated instrument index, scale and cyclic corrections.

# 6. Calibrating a baseline against a prescribed distance meter

## Mathematical model for the calibration of baselines.

**Dij = Xj –Xi – a0**  (6.1)

|  |  |  |
| --- | --- | --- |
| Where | Dij =  Xi =  Xj =  a0 = | Distance measured with a prescribed EDM instrument between pillars i and j  Distance from the first pillar to pillar i along the line running a horizontal reference height between the first and last pillar of the baseline.  Distance from the first pillar to pillar j along the line running at horizontal reference height  Additive constant of the prescribed EDM instrument. |

## Observation equations

Assume that n distances are measured with an EDM instrument on a baseline. An observation equation of the following form for each of the measured distances can be derived.

**Vij = Xj - Xi – a0 – [D ij – (Xoj - Xoi )]** (6.2)

|  |  |  |
| --- | --- | --- |
| Where | Xoi  =  Xoj  =  XI =  Xj =  Dij =  vij =  a0 = | Original baseline distance XI  Original baseline distance Xj  Correction to Xoi  Correction to Xoj  Measured distance by the prescribed EDM instrument between pillar i and j  Residual of the distance between pillar i and j  Additive constant of the prescribed EDM instrument. |

## Example: Formation of observation equations

Consider the following baseline consisting of 4 pillars

X4

X3

X2

Pillar 1 Pillar 2 Pillar 3 Pillar 4

The original baseline distances are as follows

|  |  |  |
| --- | --- | --- |
| From Pillar  (i) | To Pillar  (j) | Original baseline  Distance (Xoj) |
| 1  1  1 | 2  3  4 | Xo1 = 0.000  Xo2 = 100.000  Xo3 = 200.000  Xo4 = 300.000 |

The following measurements have been carried out with an prescribed EDM instrument.

|  |  |  |
| --- | --- | --- |
| From Pillar  (i) | To Pillar  (j) | Measured Distance  (Djj) |
| 1  1  1  2  2  3 | 2  3  4  4  3  4 | D12 = 100.010  D13 = 200.015  D14 = 300.020  D24 = 200.010  D23 = 100.010  D34 = 100.005 |

The observation equations as per equation 6.2 are:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| V12 =  V13 =  V14 =  V24 =  V23 =  V34 = | X2  X2  X2 | X3  X3  X3 | X4  X4  X4 | -a0  -a0  -a0  -a0  -a0  -a0 | -[D 12 - (Xo2 - Xo1 )]  -[D 13 - (Xo3 - Xo1 )]  -[D 14 - (Xo4 - Xo1 )]  -[D 24 - (Xo4 - Xo2 )]  -[D 23 - (Xo3 - Xo2 )]  -[D 34 - (Xo4 - Xo3 )] |

The observation equations for all distances measured on a calibration baseline can be further expressed in a matrix form as **V = AX + W**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Where A = | -1  0  0  1  1  0 | 0  -1  0  0  -1  1 | 0  0  -1  -1  0  -1 | -1  -1  -1  -1  -1  -1 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| V = | V12  V13  V14  V24  V23  V34 | X = | X2  X3  X4  a0 | W = | -[D 12 - (Xo2 - Xo1 )] = 0.010  -[D 13 - (Xo3 - Xo1 )] = 0.015  -[D 14 - (Xo4 - Xo1 )] = 0.020  -[D 24 - (Xo4 - Xo2 )] = 0.010  -[D 23 - (Xo3 - Xo2 )] = 0.010  -[D 34 - (Xo4 - Xo3 )] = 0.005 |

## Least squares solution for the corrections to the baseline distances

The equation used for least squares solution of the baseline distances corrections is:

**X = - (ATPA)-1 ATPW** (6.3)

|  |  |  |
| --- | --- | --- |
| Where | A =  P =  W = | Matrix of coefficients (equation 5.7)  Weight Matrix (equations 5.10 and 5.11)  Misclose Vector (equation 5.8) |

The values for the weight matrix (P) can be determined according to the procedures outlines in section 4.

# 7. Analysis of the least squares results

## 7.1 Variance-covariance matrix of the parameters

The cofactor matrix of the parameters is:

**QXX = S2 (ATPA)-1** (7.1)

|  |  |  |
| --- | --- | --- |
| Where | A =  P = | Matrix of coefficients of the unknown parameters (observation equations)  Diagonal Weight Matrix of the observations |

The a posteriori variance factor S2 is calculated from the least squares solution

 (7.2)

 (7.3)

|  |  |  |
| --- | --- | --- |
| Where | v =  s = | Residual between the fixed distance and adjusted distance  Standard deviation of the measured line |

d = degrees of freedom = n – u (7.4)

|  |  |  |
| --- | --- | --- |
| Where | n =  u = | Number of observed lines  Number of parameters (a0,a1,b11,b12,b21,b22) = 6 |

The experimental standard deviation of a single measured distance can be calculated as per clause 6.3 in the Standard Australia document ISO 17123-4:2001

Experimental standard deviation of a single measured distance =  (7.5)

The use of the experimental standard deviation of a single measured distance is of limited value in analysing the calibration of an EDM instrument or baseline and is therefore not used in the Baseline software.

The estimated variance-covariance matrix of the parameters X is

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| QXX = | a0 a0 | a0 a1  a1 a1 | a0 b11  a1 b11  b11 b11 | a0 b12  a1 b12  b11 b12  b12 b12 | a0 b21  a1 b21  b11 b21  b12 b21  b21 b21 | a0 b22  a1 b22  b11 b22  b12 b22  b21 b22  b22 b22 | (7.6) |

The a posteriori standard deviation of the instrument index, scale correction and cyclic errors can be derived by taking the square root of the diagonal elements in the above equation. For example the standard deviation of the instrument correction 

A 95% confidence level is obtained by multiplying the standard deviation by t95%,d where t stands for the Student’s t-distribution (Refer to Section 7.4)

## 7.2 The effect of the uncertainties of meteorological instruments on the a posteriori uncertainty of the baseline.

The meteorological instruments (Psychrometers, Barometers, Humidity sensors) used for the calibration of the certified distances of a baseline are generally calibrated. The uncertainties of these calibrated instruments should be added to the least squares computed uncertainty of the baseline using the General Law of Propagation of Variances. This may significantly increase the scale component of the least squares computed uncertainty of the baseline.

## 7.3 Statistical significance of any of the unknown parameters

The statistical significance of the least squares estimated instrument index, scale and cyclic errors can be checked by the following equation

Test statistics T:

 (7.7)

Where t stands for the Student’s t-distribution and qXX for the diagonal element of the cofactor matrix of the parameters QXX which corresponds to the Xi to be tested. A two tailed test is used for the derivation of the Student’s t-distribution.

This test is primarily used to check the significance of the cyclic errors. For most EDM instruments the cyclic errors are insignificant and are therefore not included in the instrument correction.

Equation 7.7 can be simplified for testing the significance of the cyclic errors:







 (7.8)

If the test passes for even one of the cyclic errors then all cyclic errors are considered significant and therefore should be included in the instrument correction. If the test fails for all the cyclic errors then all cyclic errors are insignificant and should not be included in the instrument correction. The above test is not used for the instrument index and scale corrections as these should always be included in the EDM calibration report even if they are zero.

## 7.4 Minimum Standards for uncertainty of EDM instruments.

Recommendation No.8 of the Working party of the National Standards Commission on the “calibration of E.D.M. Equipment” specifies that the 95% confidence level of the instrument correction shall not exceed:

 (7.9)

The uncertainties of a0 and a1 are expressed at the 95% confidence level

This recommendation means that an instrument correction is derived for a distance meter/reflector combination from measurements on a certified EDM baseline and that the uncertainty (against the National Standards) of this instrument correction (IC) shall not exceed 4.0 mm + 20ppm.

The uncertainty of an instrument correction at the 95% confidence level for a particular distance (D) is :

 (7.10)

|  |  |  |
| --- | --- | --- |
| Where | fT =  f =  QfXX =  D = | [ 1.0, 0.001D, 1.0, 1.0]  transpose of row matrix fT  sum-matrix of the variance-covariance matrix of the unknown parameters (QXX) comprising all variance and co-variances between the parameters a0,a1,b12 andb22.  Distance in metres |

Test for checking the instrument uncertainties over a given distance.



In program “BASELINE” this test is carried out for distances of 50, 250, 500, 750, 1000 and 2000 metres.

In the following example of an EDM calibration report produced by program “BASELINE” the uncertainty of instrument constant (a0 + a1) = 3.1- mm + 7.97 ppm at the 95% confidence level was computed from the equations in section 7.1. The uncertainty of the instrument constant over specified distances have been calculated by using equation 7.9

Example of an EDM calibration showing the uncertainty of an instrument correction

|  |
| --- |
| **UNCERTAINTY OF THE INSTRUMENT CONSTANT AGAINST PRESCRIBED STANDARDS**  Minimum standard for the uncertainty of calibration of an EDM instrument is 4.00 mm + 20 ppm as described in terms of Recommendation No.8 of the Working Party of the National Standards Commission on the calibration of E.D.M. Equipment of 1 February, 1983. All uncertainties are specified at 95% confidence.    Uncertainty of instrument constant: 0.86 mm + 1.94 ppm    INSTRUMENT MINIMUM  DISTANCE UNCERTAINTY STANDARD COMPARISON  (metres) (mm) (mm) TEST  -------- ----------- ---------- -----------  50 0.92 5.00 PASS  100 0.83 6.00 PASS  200 0.68 8.00 PASS  300 0.58 10.00 PASS  400 0.55 12.00 PASS  600 0.73 16.00 PASS  The uncertainty of the instrument constant satisfies the National Standards Commission recommended minimum standard where a 'PASS' is indicated. |

## 7.5 Least Uncertainty of Measurement (LUM))

The LUM is usually set during an accreditation assessment of a baseline. The verifying authority responsible for the calibration of a baseline can not claim a better uncertainty than the LUM. In the calibration reports the LUM instead of the actual computed uncertainty will be displayed if the actual computed uncertainty is smaller than the LUM. If the baseline calibrations regularly achieves better than the current LUM, then at the next accreditation assessment, consideration could be given to revise the LUM to the improved figure.

## 7.6 Student’s T-distribution (t,d)

The following equations are used to compute the Student’s t-distribution (t,d) for a given degree of freedom (d) and a desired confidence level () . These equations were extracted from Pope’s tau subroutine as written in the NOAA Technical Report NOS65 NGS1, :The Statistics of Residuals and the detection of outliers”, May 1976. Alternatively values for t,d can be derived from pertinent statistical tables.

Probability value (P) = Confidence Level (%)/100

= 1-P, For a two tailed test  = (1-P)/2

d = degrees of freedom

F = 1.3862943611199 – 2 log(





Y = X2











Student’s T distribution value (t,d) = X + V(A+V(B+V(C+EV))) (7.11

## 7.7 Tests on the variance factor

 (7.12)

|  |  |  |
| --- | --- | --- |
| Where | D=  =   = | Degrees of freedom  Significance level for statistical testing (eg. 0.05 = 95% confidence level)  Chi-square distribution value (derived from tables or subroutines) |

The Chi-square test on the variance factor can be used as a reliable statistical test for determining if the observations have been correctly weighted provided that there are no gross errors in the observations.

## 7.8 Detecting gross and systematic errors

Gross and systematic errors are detected by analysing the residual and standardised residuals. Standardised residuals are more effective than residuals for the location of gross errors

v = D – (d + IC) (7.13)

|  |  |  |
| --- | --- | --- |
| Where | v=  D =  d =  IC = | Residual  Fixed distance between pillars.  Measured reduced distance.  Instrument corrected (Sum of the least squares estimated index, scale and cyclic corrections) |

The standardised residual is the residual divided by its standard deviation

 (7.14)

Where vi is the a posteriori standard deviation of the residual. vi can be calculatedfrom the cofactor matrix of the parameters QXX and the a priori weight matrix P.

vv = P-1- L (7.15)

|  |  |  |
| --- | --- | --- |
| Where | L=  VV= | Variance-covariance matrix of the adjusted quantities = A QXX AT  Variance- covariance matrix of the residuals. |

If the observations contain only random errors and we have made correct assumptions for the input weights than the standardised residuals will follow the familiar shape of the Normal distribution frequency curve. A histogram comparing the standardised residuals with the Normal distribution curve will facilitate the detection of observational errors.

Observations with very large residuals should be rejected, however for the smaller standardised residuals you have to decide whether they are caused by:

* Large random errors, in which case you should retain the observation.
* Systematic errors, in which case it should be rejected.

A value known as the rejection criterion can be computed which can be used to flag observations for possible rejection when their standardised residuals exceed this value. This criterion should reduce the possibility of rejecting good observations (type I errors) and of leaving systematic errors in the data (type II) errors).

Standardised residual < Residual rejection criterion (7.16)

By increasing the rejection criterion you reduce the possibility of rejecting good observations but increase the possibility of leaving gross errors in the data. The residual rejection criterions are generally based on statistical functions and are sensitive to the degrees of freedom and the number of observations. A confidence level must be specified to determine the probability of detecting gross errors. A considerable number of methods exist for the computation of this criterion. One of the most suitable methods for computing a residual rejection criterion is the procedure known as the “tau” criterion described by Allen J.Pope in the “NOAA Technical report NOS 65 NGS1”

 (7.17)

|  |  |  |
| --- | --- | --- |
| Where | c =  =  n =   = | Residual rejection criterion  Value of the tau distribution (derived from tables or subroutines supplied by Allen J.Pope)  Number of observations  Significance level |
| Confidence level = 100 (1-) % | |

The software application “BASELINE” does not automatically calculate the residual rejection criterion. However the user can specify any rejection criterion. The default rejection criterion in “BASELINE” is three standard deviations. The observed lines containing standardised residuals, which exceed the specified residual rejection criterion are flagged by asterisks in the calibration report.

## 7.9 Interlaboratory comparison

# Reference

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# ISO:….

# APPENDIX A

**Example of an EDM Instrument Calibration Certificate**

# APPENDIX B

**Example of a Baseline Calibration Certificate**

# APPENDIX C

**Example of an EDM Instrument Calibration Report**

# APPENDIX D

**Examples from a full Baseline Calibration Report**